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# The Common–Trend and Transitory Dynamics in Real Exchange Rate Fluctuations\*

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## Abstract

This study explores the sources of real exchange rate fluctuations under the current float. Using a cointegration model of the real exchange rate, the innovations are decomposed into transitory and common–trend components. Both transitory and common–trend innovations are found to explain an appreciable portion of real exchange rate fluctuations, albeit their relative importance can vary across major currencies. Further analysis suggests that common–trend innovations are attributable to both productivity and monetary changes, albeit transitory innovations are linked primarily to monetary changes. The empirical results are largely consistent with an open–economy macroeconomic model.

JEL: F31, F41

KEY WORDS: Real exchange rate; common trend; productivity shock; monetary shock.

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# 1 Introduction

Although international economists have commonly recognized the existence of large, persistent deviations from purchasing power parity (PPP) during the current float, identifying the contributing role of economic fundamentals in driving the deviations remains an unsettled empirical issue. The PPP arbitrage condition, which serves as a key building block for many monetary models of exchange rate determination, suggests a long-run equilibrium relationship between exchange rates and relative national price levels. The analytical simplicity of the PPP relationship, as Lothian (1997) observes, may have contributed to its popular use.

The PPP relationship can be viewed as the open-economy extension of the quantity theory of money. It posits that monetary disturbances have no permanent impact on the real exchange rate and that real disturbances exist but they do not preclude long-run reversion toward PPP.

Early studies of the current float experience generally report evidence of a unit root in the real exchange rate (see Froot and Rogoff (1995) for an excellent survey), implying that permanent shocks are the predominant source of real exchange rate movements and that theoretical or empirical modeling of the underlying determinants of PPP deviations should focus primarily on real factors. An increasing number of recent studies, however, revive the empirical support for PPP reversion and find no unit root in real exchange rates during the recent float (Frankel and Rose, 1996; Oh , 1996; Papell, 1997; Wu, 1996; Cheung and Lai, 1998; Taylor and Sarno, 1998; Culver and Papell, 1999, Kuo and Mikkola (1999)). These findings of parity reversion suggest that economic models emphasizing real factors as the principal determinants of real exchange rates will seriously understate the importance of monetary shocks. On the other hand, the stationarity findings should not deny the relevance of real shocks entirely. Real shocks may still contribute to the observed persistence in real exchange rates. Indeed, the empirical rate of PPP reversion seems too sluggish to be explained by purely nominal shocks (Rogoff, 1996).

Several studies have assessed the contribution of monetary shocks to dollar-based real exchange rates using the vector autoregression approach. Clarida and Gali (1994) and Eichenbaum and Evans (1995) analyze real exchange rate dynamics — including quarterly real rates of British pound, German mark and Japanese yen — during the current float, while Rogers (1999) examines over 100 years of annual data on real British pound rates. Although the contribution estimates have been mixed and the relative importance of monetary and real influences on real exchange rates is still in dispute, many models of exchange rate determination focus on monetary shocks exclusively (Beaudry and Devereux, 1995; Obstfeld and Rogoff, 1995a; Alvarez and Atkeson, 1997; Chari, Kehoe and McGrattan, 1998).

Canzoneri, Cumby and Diba (1999) study a panel of 13 OECD countries and report that relative prices between traded and nontraded goods are cointegrated with relative labor productivity. In analyzing a long historical series of the real pound/dollar rate, Engel

(2000) affirms a significant role of real shocks and notes that standard unit-root tests cannot conclusively rule out the presence of a permanent component when the transitory component is much more volatile.

Engel and Kim (1999) consider the real pound rate to be nonstationary, nevertheless, and they decompose its dynamics into transitory and permanent components using a univariate Kalman filter model. While the transitory component is found to be linked to monetary factors, the permanent component is cointegrated with relative per capita real output. The analysis follows the conventional dichotomy between the roles of real and monetary shocks: Real shocks generate permanent effects, whereas monetary shocks produce purely temporary effects. Such a dichotomous role ascription seems unnecessarily stringent and may not be totally accurate. In particular, real changes can produce purely temporary effects (Evans and Lothian, 1993). Monetary changes, on the other hand, can generate permanent effects through intertemporal smoothing of traded goods consumption or cross-country wealth redistribution effects (Rogoff, 1992; Obstfeld and Rogoff, 1995a). Lastrapes (1992) and Enders and Lee (1997) also assume that the real exchange rate is nonstationary and find that real shocks tend to dominate. Hence, the actual impact of real and monetary changes should remain open as an empirical issue.

Taking a departure from nonstationary models, this study explores an alternative perspective on the individual roles of productivity and monetary changes under stationary real exchange rates. Instead of studying univariate series of real exchange rates, we examine the joint behavior of nominal exchange rates and relative prices using a vector error correction (VEC) model. Even if the exchange rate and the relative price converge to an equilibrium relationship over the long run, the two variables can exhibit rather different short-term behavior. Hence, the bivariate VEC model may capture PPP adjustment with richer dynamics than an univariate real exchange rate model.

When the real exchange rate is stationary, the joint dynamics of the nominal exchange rate and the relative price can be decomposed into a common-trend component and a transitory component, thereby allowing us to identify the possibly different sources of disturbances driving real exchange rate dynamics. We then extract these two components and determine whether they are ascribable to macroeconomic changes in real and monetary factors. This allows us to gain more insights into the driving forces of real exchange rates. Our study finds that common-trend innovations are attributed to both productivity and money supply changes. Hence, productivity changes influence real exchange rate fluctuations, without requiring nonstationarity in the real exchange rate.

It should be noted that common-trend innovations cannot be identified in a univariate model of the real exchange rate, they cannot be separately extracted from real exchange rate series. This underscores the special merit of our bivariate modeling approach. Innovations to the common-trend component are permanent shocks to the exchange rate and the relative price, but the long-run responses of these variables will offset one another, leaving no permanent effects on the real exchange rate. The real exchange rate

thus maintains its stationarity, while common-trend innovations, along with transitory innovations, drive the dynamics of the real exchange rate — including its variability and persistence.

## 2 An Open-Economy Macroeconomic Model

A simple consumption smoothing model of the current account is examined (see, e.g., Obstfeld and Rogoff (1995b, 1996)). Let  $c_{T,s}^j$  and  $c_{N,s}^j$  be the consumption of traded and nontraded goods in period  $t = s$ , and let  $P_{T,t}^j$  and  $P_{N,t}^j$  be their respective prices (the superscript  $j = d$  for the home country and  $j = f$  for the foreign country). Let  $\frac{1}{1+\phi}$  denote the time-preference factor. The representative consumer maximizes

$$U_t^j = \sum_s^t \left( \frac{1}{1+\phi} \right)^{s-t} E_t [\theta \ln(c_{N,s}^j) + (1-\theta) \ln(c_{T,s}^j)] \quad j = d \text{ and } f \quad (1)$$

subject to an intertemporal budget constraint expressed in standardized units of tradables:

$$CA_t^j = b_{t+1}^j - b_t^j = y_t^j - c_{T,t}^j - \pi_t^j - r_t b_t^j - g_{T,t}^j - \pi_t^j g_{N,t}^j \quad (2)$$

where  $CA_t^j$  indicates the current account balance over period  $t$ ,  $b_t^j$  is the net foreign asset holdings at time  $t$ ,  $y_t^j$  is the total output,  $\pi_t^j = P_{N,t}^j/P_{T,t}^j$  gives the relative price of nontraded to traded goods,  $r_t b_t^j$  is the interest earned on foreign assets in period  $t$  (the world interest rate,  $r$ , is given as fixed for simplicity), and  $g_{T,t}^j$  and  $g_{N,t}^j$  denote the government consumption of traded and nontraded goods. Firms choose inputs of capital ( $K_{T,t}^j$  or  $K_{N,t}^j$ ) and labor ( $L_{T,t}^j$  or  $L_{N,t}^j$ ) to produce goods using the following production functions:

$$y_{T,t}^j = A_{T,t}^j (K_{T,t}^j)^m (L_{T,t}^j)^{1-m} \quad j = d \text{ and } f \quad (3)$$

$$y_{N,t}^j = A_{N,t}^j (K_{N,t}^j)^m (L_{N,t}^j)^{1-m} \quad j = d \text{ and } f \quad (4)$$

where  $A_{T,t}^j$  and  $A_{N,t}^j$  capture productivity shifts and where  $y_{T,t}^j$  and  $y_{N,t}^j$  give the corresponding output of traded and nontraded goods. The total output is measured by  $y_t^j = y_{T,t}^j + \pi_t^j y_{N,t}^j$ . For nontraded goods, their supply equals their total domestic consumption under the market-clearing condition:

$$y_{N,t}^j = c_{N,t}^j + g_{N,t}^j \quad j = d \text{ and } f \quad (5)$$

The first-order optimality conditions for traded and nontraded goods consumption imply that

$$\pi_t^j = P_{N,t}^j/P_{T,t}^j = \omega_j c_{T,t}^j/c_{N,t}^j \quad j = d \text{ and } f \quad (6)$$

where  $\omega_j = \theta_j/(1 - \theta_j)$ . Let the real exchange rate be denoted by  $Q_t$  and defined as

$$Q_t = \mathcal{E}_t P_t^f / P_t^d \quad (7)$$

where  $\mathcal{E}_t$  is the price of foreign currency in units of domestic currency and  $P_t^f$  and  $P_t^d$  are the general price levels in the relevant countries. These general price levels are considered to be given by

$$P_t^j = (P_{N,t}^j)^\theta (P_{T,t}^j)^{1-\theta} \quad j = d \text{ and } f \quad (8)$$

Assuming that the PPP relationship holds for the relative price of tradables over the long run, we have

$$\zeta_t = \mathcal{E}_t P_{T,t}^f / P_{T,t}^d \quad (9)$$

where the PPP deviation,  $\zeta_t$ , is stationary and mean-reverting. Using (8) and (9), we can rewrite (7) as

$$Q_t = \left( \pi_t^f / \pi_t^d \right)^\theta \zeta_t. \quad (10)$$

Combining equations (5) and (6) further yields

$$\pi_t^j = \omega_j (1 - \xi_t^j)^{-1} c_{T,t}^j / y_{N,t}^j. \quad (11)$$

with  $\xi_t^j = g_{N,t}^j / y_{N,t}^j$  being the share of government spending in the output of nontraded goods. If we simplify by assuming a stable ratio of government spending to output, equations (7), (10), and (11) will imply that

$$\Delta \ln Q_t = \Delta \ln \mathcal{E}_t - \Delta \ln \frac{P_t^d}{P_t^f} = \Delta \ln \zeta_t + \theta \left\{ \Delta \ln \frac{c_{T,t}^f}{c_{T,t}^d} - \Delta \ln \frac{y_{N,t}^f}{y_{N,t}^d} \right\} \quad (12)$$

where  $\Delta$  is the standard difference operator. This indicates that shocks to the real exchange rate may be viewed as structural disturbances influencing either  $\zeta_t$  or  $c_{T,t}^f / c_{T,t}^d$  or  $y_{N,t}^f / y_{N,t}^d$ . While the short-term dynamics of  $\zeta_t$  can be influenced by monetary factors such as relative interest rates (Dornbusch, 1976),  $c_{T,t}^f / c_{T,t}^d$  and  $y_{N,t}^f / y_{N,t}^d$  are driven by real factors. Specifically,  $y_{N,t}^f / y_{N,t}^d$  is a function of productivity factors, as indicated by (4). Under balanced growth, the equilibrium capital/labor ratio is constant and we then have

$$\Delta \ln \left( \frac{y_{N,t}^f}{y_{N,t}^d} \right) = \Delta \ln \left( \frac{A_{N,t}^f}{A_{N,t}^d} \right) - \Delta \ln \left( \frac{L_{N,t}^f}{L_{N,t}^d} \right). \quad (13)$$

Indeed, with no capital accumulation in our simplified model,  $\Delta \ln (K_{N,t}^f / K_{N,t}^d) = 0$  so that  $\Delta \ln (L_{N,t}^f / L_{N,t}^d) = 0$  in equilibrium. Changes in  $y_{N,t}^f / y_{N,t}^d$  can thus be explicitly linked to relative productivity changes.

Empirically, the real exchange rate is often measured based on general price levels,

as in equation (7) where the nominal exchange rate and the relative price are assumed to be nonstationary variables. If these two variables are cointegrating such that  $\ln Q_t$  is mean-reverting then  $\ln \mathcal{E}_t$  and  $\ln P_t^d/P_t^f$  are driven by a common stochastic trend. This common trend can be driven by real or monetary factors. To illustrate, we consider the demand for money (denoted by  $M_{D,t}^j$ ) in each country to be a function of the output of traded and nontraded goods and their prices:

$$M_{D,t}^j = M_D^j (P_{T,t}^j, P_{N,t}^j, y_{T,t}^j, y_{N,t}^j) \quad (14)$$

Interest rates may be included also in the money demand function without altering the analysis. In monetary equilibrium, the money demand equals the money supply ( $M_{S,t}$ ). In the absence of money illusion, the money demand function should be homogeneous of degree one in prices so that

$$M_{S,t}^j = P_t^j M_D^j (\tau_{T,t}^j, \tau_{N,t}^j, y_{T,t}^j, y_{N,t}^j) \quad (15)$$

where  $\tau_{T,t}^j = P_{T,t}^j/P_t^j$  and  $\tau_{N,t}^j = P_{N,t}^j/P_t^j$ , with  $P_t^j$  being the general price level. In view of (6) and (8), we also have  $\tau_{T,t}^j = (\pi_t^j)^\theta$  and  $\tau_{N,t}^j = (\pi_t^j)^{1-\theta}$ . Hence, it follows from (15) that

$$\ln \left( \frac{P_t^d}{P_t^f} \right) = \ln \left( \frac{M_D^f (\tau_{T,t}^f, \tau_{N,t}^f, y_{T,t}^f, y_{N,t}^f)}{M_D^d (\tau_{T,t}^d, \tau_{N,t}^d, y_{T,t}^d, y_{N,t}^d)} \right) + \ln \left( \frac{M_S^d}{M_S^f} \right). \quad (16)$$

All else being equal, a change in the domestic money supply will, when  $\ln Q_t$  is stationary, lead to proportional long-term comovements in  $\ln \mathcal{E}_t$  and  $\ln P_t^d/P_t^f$ . Also, real shocks such as productivity changes can contribute to common-trend movements in  $\ln \mathcal{E}_t$  and  $\ln P_t^d/P_t^f$  through the ratio of money demand in the two countries by influencing the relative price of traded to nontraded goods. Our analysis, thus, suggests that both productivity and monetary changes can contribute to the common-trend dynamics generating fluctuations in the real exchange rate even under the assumption that the real exchange rate is stationary.

### 3 The Statistical Framework

Let us now assume that the joint behavior of the logarithm of the nominal exchange rate ( $e_t = \ln \mathcal{E}_t$ ) and the logarithm of relative price levels ( $p_t = \ln P_t^d/P_t^f$ ) can be modeled as the following bivariate VEC system:

$$\Delta X_t = \mu + \sum_{j=1}^k \Gamma_j \Delta X_{t-j} - \Pi X_{t-k} + \varepsilon_t \quad (17)$$

where  $X_t = [e_t \ p_t]'$ ,  $-\Pi = \alpha\beta'$ ,  $\text{rank}(\Pi) = 1$ ,  $\alpha = [\alpha_1 \ \alpha_2]'$  contains adjustment coefficients,  $\beta$  is the cointegration vector, and  $\varepsilon_t$  are i.i.d. with mean zero and covariance



matrix,  $\Omega$ . Under our maintained assumption of cointegration, the system innovations can be decomposed into a common permanent component and a transitory component. Furthermore, imposing the long-run PPP condition,  $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$ , the real exchange rate, given by  $\beta'X_t$ , is stationary. Under this assumption, permanent innovations to  $e_t$  and  $p_t$  will have no long-run effects on the real exchange rate. If no cointegration exists, then there are two permanent components (and no temporary component) and permanent innovations to  $e_t$  and  $p_t$  will have long-run effects on the real exchange rate.

Since  $\Delta X_t$  is stationary, the Wold decomposition theorem implies the presence of a vector moving average (VMA) representation:

$$\Delta X_t = \delta + C(L)\varepsilon_t \quad (18)$$

where  $L$  is the lag operator,  $C(L) = I + \sum_{k=1}^{\infty} C_k L^k$ ,  $I$  is the  $2 \times 2$  identity matrix,  $\text{rank}[C(1)] = 1$ ,  $\beta' C(1) = 0$ ,  $\delta = C(1)\mu$ , and  $\beta'\delta = 0$ . Equations (17) and (18) allow us to study the short- and long-run interactions between the exchange rate and the relative price and to analyze the impulse responses to variable-specific shocks (Lütkepohl and Reimers, 1992; Pesaran and Shin, 1998).

The common-trend (CT) representation of a cointegrated system, on the other hand, can be used to investigate the impact of shocks of a specific nature: shocks to the transitory component as opposed to shocks to the CT component. The CT representation for the VEC model is (Stock and Watson, 1988):

$$X_t = \mu_0 + \Phi\eta_t + C^*(L)w_t \quad (19)$$

where  $\eta_t = \rho + \eta_{t-1} + \varphi_t$ ;  $C^*(L)$  is a stationary lag polynomial and  $w_t = \begin{bmatrix} \varphi_t & \psi_t \end{bmatrix}'$  with  $\varphi_t$  being the shock to the CT component and  $\psi_t$  being the innovation to the transitory component ( $\varphi_t$  and  $\psi_t$  are sometimes referred to as structural innovations). Equivalently, we have

$$X_t = \mu_0 + \Phi \{ \eta_0 + \rho t + \sum_{i=1}^t \varphi_i \} + C^*(L)w_t \quad (20)$$

The common stochastic trend,  $\eta_t$ , determines the trending behavior of the exchange rate and the relative price through the loading matrix,  $\Phi$ . The transitory dynamics of the system are governed by  $C^*(L)w_t$ . In addition, since cointegration implies that  $\beta'\Phi = 0$ , the dynamics of the real exchange rate are given  $\beta'\mu_0 + \beta'C^*(L)w_t$ .

Note that the CT representation is linked to the VMA representation since we can rewrite  $C(L)$  as  $C(1) + (1 - L)C^*(L)$ . Furthermore, let the transformation matrix  $F$  link the estimated residuals in  $\varepsilon_t$  to the structural shocks in  $w_t$ ,  $w_t = F\varepsilon_t$ . The structural VMA model can then be written as

$$\Delta X_t = \delta + C(L)F^{-1}w_t. \quad (21)$$

If  $F = \begin{bmatrix} F_\varphi & F_\psi \end{bmatrix}'$  can be explicitly determined, individual shocks to the CT and tran-

sitory components of the system can also be constructed. It can be shown (see appendix A for further technical details) that for the cointegrated system under study, we have

$$\varphi_t = \det(B(1))^{-1} \begin{bmatrix} -\alpha_2 & \alpha_1 \end{bmatrix} \varepsilon_t = F_\varphi \varepsilon_t \quad (22)$$

and

$$\psi_t = (\alpha' \Omega^{-1} \alpha)^{-1/2} \alpha' \Omega^{-1} \varepsilon_t = F_\psi \varepsilon_t \quad (23)$$

where  $B(1)$  is defined in the appendix. The transitory innovation,  $\psi_t$ , generates solely temporary effects on the exchange rate and the relative price, hence no permanent effects on the real exchange rate. The CT innovation,  $\varphi_t$ , generates long-lasting effects on the exchange rate and the relative price, but these effects will cancel out one another over the long run, leaving no permanent effects on the real exchange rate under our maintained assumption that  $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$ . The impulse responses of the exchange rate and the relative price to the individual innovations are given by  $C(L)F^{-1}$ , and those of the real exchange rate are given by  $\beta' C(L)F^{-1}$ .

## 4 Preliminary Data Analysis

The dynamic behavior of exchange rates and relative prices under the current float are examined. The data consist of monthly series of consumer price indices and dollar-based exchange rates over the sample period from April 1973 through December 1998. In January 1, 1999, exchange rates among the Euro countries became irrevocably fixed. We study three major currencies — the German mark, the Japanese yen, and the British pound. The price and exchange rate data are obtained from IMF's International Financial Statistics CD-ROM. All the data series are expressed in logarithms. Standard unit-root tests were performed on individual exchange rate and relative price series. In no case can the null hypothesis of a unit root be rejected. We have also tested for unit roots in first difference and found that the null of a unit root can be rejected at conventional significance levels.

We next perform tests for cointegration between the exchange rate and the relative price and unit root tests of the real exchange rate. Johansen (1991) devises cointegration tests based on the technique of reduced rank regression under the VEC setting. We apply the trace test (adjusted by Ahn and Reinsel's (1990) finite-sample correction) to the three bivariate VAR systems we study. The trace statistics were computed to be 17.48, 12.18, and 27.18 in the respective cases of Germany, Japan, and the U.K. Except for the case of Japan, these test statistics are statistically significant at the 10% level or better, rejecting the null hypothesis of no cointegration. Note, however, that we reject the null of no cointegration for Japan at approximately the 85% level. We have examined the roots of the VEC models and found that there is a large second root in the system. The second root is smaller when we impose the restriction that the real exchange rate is stationary

compared to the model without this restriction for Japan and the UK and larger for Germany. This suggests that it may be the case that the nominal exchange rate and the relative price are not cointegrated. Estimating the system in first differences we find that the roots are very small indicating that these systems easily fulfill the stationary conditions. Our interpretation of these empirical evidence is that there is no indication of  $I(2)$  trends in the data. The large second roots of the VEC models indicate that, even though our models clearly fulfill the stationarity condition, it is difficult to distinguish between a stationary and a nonstationary model. We continue our analysis using assuming that there is one cointegration vector in our systems.

Given that the rank is 1 in all models (for the moment disregarding that the trace tests suggest no cointegration for Japan), we can test whether the PPP relationship is contained in the cointegration space. Using the method proposed by Johansen and Juselius (1990) and Johansen (2000) we find that we can reject the null that the PPP relation is stationary only for Germany at the 5 percent level. The respective p-values are 0.041, 0.156 and 0.061 in the three cases.

We also apply unit-root tests on the real exchange rate, recommended by Froot and Rogoff (1995) as the most direct way to test the PPP relation. We apply the DF-GLS test (Elliot, Rothenberg and Stock, 1996) on each of the real exchange rate series. Critical values are tabulated by Elliot, Rothenberg and Stock (1996). For the parameter defining the local alternative, the use of  $-7$  is recommended. The unit-root test statistics are computed to be  $-1.894$ ,  $-0.68$ , and  $-1.65$  in the cases of Germany, Japan, and the U.K., respectively. The test statistics in the cases of Germany and the U.K. are both statistically significant at the 10% level or better, suggesting a rejection of the null that PPP does not hold in these two cases. The case of Japan, on the other hand, fails to reject the null.

In view of our results and the growing evidence in favor of no unit root in real exchange rates — including the real yen, mark, and pound rates — reported in the PPP literature, we proceed with the assumption that long-run PPP holds. It should be noted that our aim of studying the relative importance of the common-trend in the nominal exchange rate and the relative price is not dependent on our assumption that PPP holds in the long-run. On the other hand, our analysis requires that these two variables are cointegrated, an assumption that may be questionable in the case of Japan, the Johansen trace test suggests no cointegration, the Johansen-Juselius test for specific cointegration vectors (given that there is one cointegration vector in the system) could not reject the null of PPP whereas the DF-GLS test could not reject the null that PPP does not hold. It is commonly known that both cointegration and unit-root tests generally have limited power; consequently, the failure to detect stationarity does not necessarily imply nonstationarity.

## 5 The Relative Importance of the Common–Trend Component

The VEC model in (17) is estimated and its lag specification is selected based on information criteria as well as diagnostic checking of the residuals. We use three lags for Germany, four lags for Japan, and two lags for the UK. The VEC model estimates will later be used to construct the transitory and common–trend components and compute impulse responses. Since cointegration between  $e_t$  and  $p_t$  requires that at least one of the EC coefficients is non–zero (i.e.,  $|\alpha_1| + |\alpha_2| \neq 0$ ), estimates of  $\alpha_1$  and  $\alpha_2$  are reported in Table 1. In every case the EC coefficient has the correct sign. When the PPP relationship is stationary, shocks to either  $e_t$  or  $p_t$  exert no long–run effects on the real exchange rate. Some shocks can have permanent effects on the exchange rate and the relative price, but in the long run these effects offset one another exactly. Such common permanent effects are characterized by a common trend, which describes the long–run trending behavior of both the exchange rate and the relative price.

Since the bivariate system for  $\begin{bmatrix} e_t & p_t \end{bmatrix}'$  is assumed to be cointegrated, its system dynamics can be decomposed into a transitory component and a CT component, as discussed in section 3. Figure 1 displays the estimated CT components corresponding to the cases of Germany, Japan, and the UK. These common trends (indicated by solid lines) capture the trending dynamics shared by the exchange rate and the relative price (shown by broken lines in individual graphs). In particular, the common trend can track the relative price movement very closely in every case.

We next examine how the system dynamics will react to the different innovations to the transitory and CT components. Figure 2 depicts the impulse responses of the exchange rate, the relative prices, and the real exchange rate to a one–standard–deviation transitory innovation. The 95% confidence bands (shown by broken lines) are obtained from bootstrap simulations with 500 trials. In accordance with the model identification restriction, transitory innovations have only temporary effects on the exchange rate, the relative price, and the real exchange rate. In all cases the response of the exchange rate is much stronger than that of the relative price. This may reflect the behavior that price adjustment is often quite sluggish, while exchange rate adjustment is quick in response to shocks.

Figure 3 displays the impulse responses of the various variables to a one–standard–deviation CT innovation, along with their 95% confidence bands obtained from bootstrap simulations. We notice that although the CT innovation has very long–lived effects on both the exchange rate and the relative price, these effects offset one another over time. Consequently, the CT innovation produces no permanent effects on the real exchange rate, as implied by our assumption that PPP holds in the long–run. Again, the response of the exchange rate is found to be always larger in magnitude than that of the relative price.

Variance decomposition analysis is next conducted. The forecast error variance for the exchange rate, the relative price, and the real exchange rate is broken into portions explained by the two types of innovations. The decomposition reveals the relative importance of transitory and CT innovations in explaining the fluctuations of the economic variables over different horizons (see figures 4 and 5).

Table 2 reports the forecast error decomposition estimates for the exchange rate (confidence interval estimates have been plotted in the graphs and are not tabulated to conserve space). At the short three-month horizon, transitory innovations are more significant than CT innovations in explaining the exchange rate fluctuations for the mark, but the reverse is true for the yen and the pound. The relative importance of CT innovations is particularly prominent in the yen case. Over horizons of six months or more, CT innovations consistently account for most of the exchange rate fluctuations. Indeed, the relative importance of CT innovations grows steadily as the horizon lengthens. At the four-year horizon, for instance, CT innovations can explain 65% to 88% of the exchange rate fluctuations for different countries. At the eight-year horizon, they account for 70% to 90% of the fluctuations.

The decomposition estimates for the relative price are given in Table 3. Again, different results across countries can be seen at short horizons. For the UK, transitory innovations produce a larger part of the relative price fluctuations than CT innovations over horizons of six months or less. For Japan, transitory innovations contribute to most of the relative price variation over the two-year horizon or shorter. For Germany, CT innovations, not transitory innovations, account for most of the relative price variation even over short horizons. Despite the short-horizon results, the fraction explained by CT innovations rises and the fraction ascribed to transitory innovations becomes increasingly unimportant in all the cases as the horizon extends. At the four-year (eight-year) horizon, CT innovations generate 67% to 89% (87% to 95%) of the relative price fluctuations among different countries.

Table 4 contains the decomposition estimates for the real exchange rate. Unlike the exchange rate and the relative price, for which CT innovations have burgeoning influences and always dominate at long horizons, no uniform results are found across countries for the real exchange rate. This reflects the offsetting effects of CT innovations on the exchange rate and the relative price. The contribution of CT innovations remains relatively unchanged with longer horizons for both real yen and pound rates. For the real yen rate, transitory and CT innovations explain about 29% and 71% of the real exchange rate fluctuations, respectively. For the real pound rate, the respective fractions attributable to transitory and CT innovations are estimated to be 67% and 33%. For the real mark rate, the importance of CT innovations rises and that of transitory innovations falls steadily as the horizon lengthens. Transitory innovations, nonetheless, still explain more than half of the real exchange rate variation.

In addition to the relative contributions of CT and transitory innovations to real

exchange rate variability, we analyze the persistence of real exchange rate adjustment with respect to the different innovations. The impulse–response analysis (see Figures 2 and 3) allows us to compute and compare half–life convergence speeds of the real exchange rate. We find that the real exchange rate adjustment to CT innovations is generally more persistent than that to transitory innovations, but the difference seems not substantial. For the real mark rate, the estimated half–lives are 3.8 years in response to CT innovations and 2.9 years in response to transitory innovations. For the real yen rate, the estimated half–lives are 5.0 years in the case of CT innovations and 4.5 years in the case of transitory innovations. For the real pound rate, the estimated half–lives are 2.8 years in the case of CT innovations and 2.7 years in the case of transitory innovations. These are comparable with the usually reported estimates of 3 to 5 years.

The above analysis has investigated the short– and long–horizon adjustment dynamics of the real exchange rate in a bivariate model of the exchange rate and the relative price. Our main conclusion is that both CT and transitory innovations significantly contribute to the variability of the real exchange rate and its persistence, albeit with differing relative importance across countries. It follows that empirical modeling of real exchange rate dynamics should recognize the individual significance of both CT and transitory innovations. This underscores the merit of our bivariate analysis in comparison to univariate analysis of the real exchange rate. The latter does not allow us to identify CT innovations separately.

## 6 Sources of the Common–Trend and Temporary Innovations

While CT and transitory innovations are both found to be important contributors to real exchange rate dynamics, no attempt has yet been given to study the economic transmission mechanism driving these structural innovations. Turning now to this issue, we first outline the setup for statistical testing.

Ignoring the constant term for illustrative convenience, we have from equation (21) that

$$\Delta X_t = C(L)F^{-1}w_t \quad (24)$$

where  $w_t = \begin{bmatrix} F_\varphi \varepsilon_t & F_\psi \varepsilon_t \end{bmatrix}' = \begin{bmatrix} \varphi_t & \psi_t \end{bmatrix}'$  with  $\varphi_t$  being the CT innovation and  $\psi_t$  being the transitory one (see Appendix A for the identification of  $F = [F_\varphi F_\psi]'$ ). This can be rewritten into a VAR representation:

$$A(L)\Delta X_t = F^{-1}w_t \quad (25)$$

where  $A(L) = C^{-1}(L)$  and  $A(0) = C^{-1}(0) = I$ . Let  $\Delta Y_t$  denote changes in macroeconomic

variables, and  $\Delta Y_t$  is considered to follow an ARMA process:

$$R(L)\Delta Y_t = (I - \Theta L)\xi_t \quad (26)$$

where  $R(0) = I$ . We note that a first-order MA specification is sufficient to capture the innovation dynamics of the macroeconomic variables in our study here and that more complex MA specifications can be incorporated without changing our main analysis. Let  $R^*(L) = I - R(L)$ . For the changes in macroeconomic variables to Granger-cause the CT or transitory innovation, we have

$$w_t = S(L)\Delta Y_t + u_t = G(L)\Delta Y_{t-1} + H(L)\xi_t + u_t \quad (27)$$

where  $S(L) \neq 0$ , which implies that  $G(L) = S(L)R^*(L) \neq 0$  and  $H(L) = S(L)(I - \Theta L) \neq 0$ . Running causality tests based on equation (27) directly, however, would require generated estimates of  $w_t$  and  $\xi_t$ , which are not observable variables. We would then face an errors-in-variables problem.

To avert the problem with generated regressors, equations (25) and (27) are combined to yield:

$$A(L)\Delta X_t = F^{-1}G(L)\Delta Y_{t-1} + F^{-1}H(L)\xi_t + v_t \quad (28)$$

where  $v_t = F^{-1}u_t$ . Letting  $A^*(L) = I - A(L)$  and  $R^*(L) = I - R(L)$ , equations (26) and (28) can be estimated together as a system by stacking the two equations as follows:

$$\Delta X_t = A^*(L)\Delta X_{t-1} + J_1(L)\Delta Y_{t-1} + v_t + J_2(L)\xi_t \quad (29)$$

$$\Delta Y_t = R^*(L)\Delta Y_{t-1} + \xi_t - \Theta\xi_{t-1} \quad (30)$$

where  $J_1(L) = F^{-1}G(L)$ , and  $J_2(L) = F^{-1}H(L)$ . This set of equations yields a VARMA system in  $\begin{bmatrix} \Delta X_t & \Delta Y_t \end{bmatrix}'$ , with  $\begin{bmatrix} v_t & \xi_t \end{bmatrix}'$  being the innovation vector. The lag specification can be selected using standard information criteria such as the Akaike or Schwarz criterion. Since  $G(L) = FJ_1(L)$  and  $H(L) = FJ_2(L)$ , Wald tests for causal effects of macroeconomic changes on the structural innovations in equation (27) can be conducted based on coefficient restrictions on the model estimates of  $J_1(L)$  and  $J_2(L)$  jointly. To test whether the macroeconomic variables contribute to the CT innovation, we examine the null hypothesis that  $F_\varphi J_1(L) = 0$  and  $F_\varphi J_2(L) = 0$ . To test whether the macroeconomic variables contribute to the temporary innovation, we examine the null hypothesis that  $F_\psi J_1(L) = 0$  and  $F_\psi J_2(L) = 0$ .

In the actual data analysis, we will explore whether the CT and transitory innovations are attributable to changes in both real and monetary macroeconomic variables — including changes in productivity differentials ( $\Delta PR_t$ ), relative money supply ( $\Delta MS_t$ ), and interest rate differentials ( $\Delta IR_t$ ). Accordingly,  $\Delta Y_t$  is represented by  $\begin{bmatrix} \Delta PR_t & \Delta MS_t & \Delta IR_t \end{bmatrix}'$ . Productivity is measured as industrial production divided by labor employment and hours worked (see appendix B for data sources). Long-term

interest rates are used to compute relative interest rates.

Table 5 summarizes the results from tests for causal effects of individual macroeconomic variables. Experimenting with various lag selection methods leads us to use a VARMA(3,1) model in the case of Germany, a VARMA(4,1) model in the case of Japan, and a VARMA(2,1) model in the case of the U.K. Various lags have also been tried, and the statistical results are not sensitive to the lag length. The Wald test statistics for Granger causality of individual macroeconomic variables are reported, along with their corresponding p-values. In the case of Germany, we find statistically significant evidence to support that CT innovations are linked to both productivity and money supply changes, whereas transitory innovations can be ascribed to interest rate changes. In the case of Japan, CT innovations can also be linked to both productivity and money supply changes, but transitory innovations seem not directly attributable to any of the macroeconomic variables. The U.K. case gives somewhat different results: both CT innovations and transitory innovations are linked primarily to money supply changes.

The overall results suggest that CT innovations are ascribable to productivity and money supply changes and that transitory innovations are attributable to money supply and interest rate changes. Accordingly, CT dynamics can be driven by changes in both real and monetary variables, whereas transitory dynamics are governed by changes in monetary variables mainly.

Some remarks concerning the PPP puzzle (Rogoff, 1996) discussed in the recent literature should be noted. Although the enormous short-term volatility of the exchange rate suggests a likely important role of monetary shocks, the observed half-life persistence of the real exchange rate seems too high to be generated by monetary disturbances, even allowing for price stickiness. Rogoff (1996) discusses the idea of a middle ground that the real exchange rate is buffeted by both monetary and real shocks. If this mixed-shocks explanation is valid, PPP reversion should actually be rather fast subsequent to monetary shocks, and the high persistence in the real exchange rate would largely be induced by real shocks.

Our findings generally confirm the significant contributions of both monetary and productivity changes to CT dynamics. However, the allowance for the productivity effect does not help solve the PPP puzzle. In view of the half-life estimates reported earlier, the persistence of the real exchange rate with respect to transitory innovations remains too high (with half-lives ranging from 2.7 to 4.5 years). Given that transitory innovations are shown to come primarily from monetary changes, productivity changes play little role in generating such highly persistent dynamics in this case. In other words, the PPP puzzle cannot be resolved by identifying the relative importance of monetary and productivity shocks. The mixed-shocks explanation does not fully solve the puzzle. Interestingly, Rogoff (1992), Obstfeld and Rogoff (1995), and Chari, Kehoe and McGrattan (1998) have demonstrated in theory that monetary shocks can generate persistent real exchange rate dynamics.



With monetary and productivity factors being found to contribute to CT dynamics, on the other hand, our results do support the importance of not only monetary factors but also real factors in real exchange rate fluctuations. Indeed, the empirical findings underline the need to recognize the significant influences of both monetary and productivity factors on the real exchange rate in more structural modeling of its dynamics. The relevance of both factors also accords with the theoretical open-economy model. The important point here is that even with long-run PPP considered to hold, productivity effects should still be included in structural equations of real exchange rate determination.

## 7 Summary and Concluding Remarks

The contribution of the common-trend dynamics in real exchange rate fluctuations has not been analyzed in previous studies. Taking a first step, this study devises a statistical scheme to decompose real exchange rate dynamics into transitory and common-trend innovations based on a cointegration model. It is shown that both transitory and common-trend innovations are responsible for a significant portion of real exchange rate fluctuations, albeit their relative importance may vary across currencies. Common-trend innovations are more important to the real Japanese yen rate, whereas transitory innovations are more important to the real German mark and British pound rates.

Theoretically, monetary and productivity changes can both influence real exchange rate dynamics, as illustrated in an open-economy macroeconomic model. In consistent with the theoretical result, our empirical findings support that common-trend innovations is attributable to monetary and productivity changes alike, albeit transitory innovations are linked mainly to monetary changes. We further observe that the empirical relevance of productivity shocks in real exchange rate fluctuations is established without imposing nonstationarity in the real exchange rate. The results here thus complement the growing evidence of PPP reversion in real exchange rates and reinforce the emerging evidence for the significance of productivity shocks reported in prior studies.

Some final remarks on the bivariate VEC modeling of the real exchange rate are in order. To determine the contributing role of productivity changes in PPP deviations, the standard empirical approach in the literature is to analyze univariate series of the real exchange rate, estimate its permanent and transitory components, and then link them to different economic factors. Presumably because productivity changes are treated as real changes impacting relative prices, the conventional wisdom is that productivity factors should be associated with the permanent component, not the transitory component. The permanent component cannot be identified, however, unless PPP deviations are nonstationary and no long-run reversion exists. Consequently, previous PPP studies of the productivity effect commonly argue for the existence of a unit root in the real exchange rate, although many other recent studies on real exchange rate dynamics have found no unit root.

Using the bivariate VEC model of the real exchange rate to identify the productivity effect represents a significant departure from the conventional wisdom. On the analytical level, productivity changes, like monetary changes, can have persistent effects on relative prices without causing nonstationarity in PPP deviations. Indeed, PPP deviations are not permanent under the VEC model, but productivity changes still contribute significantly to real exchange rate dynamics.

On the statistical modeling level, the bivariate VEC analysis permits a decomposition of the innovations to the real exchange rate into two sources: (1) innovations that have only transitory effects on the exchange rate or the relative price, and thus transitory effects on PPP deviations, and (2) innovations that have permanent effects on either variable but transitory effects on PPP deviations. Productivity disturbances show up as the latter type of innovations. This underlines the usefulness of our VEC modeling approach as opposed to univariate models of the real exchange rate, which essentially lump all transitory disturbances to PPP together and draw no distinction between the possibly different types of transitory effects.

## References

- Ahn, S. K. and G. C. Reinsel, (1990), “Estimation for Partially Nonstationary Multivariate Autoregressive Models,” *Journal of the American Statistical Association*, 85, 813–823.
- Alvarez, F. and A. Atkeson, (1997), “Money and Exchange Rates in the Grossman–Weiss–Rotemberg Model,” *Journal of Monetary Economics*, 40, 619–640.
- Beaudry, P. and M. B. Deveraux, (1995), “Money and the Real Exchange Rate with Sticky Prices and Increasing Returns,” *Carnegie–Rochester Conference Series on Public Policy*, 43, 55–101.
- Bergman, U. M., (1996), “International Evidence on the Sources of Macroeconomic Fluctuations,” *European Economic Review*, 40, 1237–1258.
- Campbell, J. Y. and R. J. Shiller, (1988), “Interpreting Cointegrated Models,” *Journal of Economic Dynamics and Control*, 12, 505–522.
- Canzoneri, M. B., R. E. Cumby and B. Diba, (1999), “Relative Labor Productivity and the Real Exchange Rate in the Long–Run: Evidence for a Panel of OECD Countries,” *Journal of International Economics*, 47, 245–266.
- Chari, V., P. J. Kehoe and E. R. McGrattan, (1998), “Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?,” Federal Reserve Bank of Minneapolis Staff Report No. 223.

- Cheung, Y.-W. and K. S. Lai, (1998), "Parity Reversion in Real Exchange Rates During the Post-Bretton Woods Period," *Journal of International Money and Finance*, 17, 597-614.
- Clarida, R. and J. Galí, (1994), "Sources of Real Exchange Rate Fluctuations: How Important are Nominal Shocks?," *Carnegie Rochester Conference Series on Public Policy*, 41, 1-56.
- Culver, S. E. and D. H. Papell, (1999), "Long-Run Purchasing Power Parity with Short-Run Data: Evidence with a Null Hypothesis of Stationarity," *Journal of International Money and Finance*, 18, 751-768.
- Dornbusch, R., (1976), "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*, 84, 1161-1176.
- Eichenbaum, M. and C. L. Evans, (1995), "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *Quarterly Journal of Economics*, 110, 975-1009.
- Elliott, G., T. J. Rothenberg and J. H. Stock, (1996), "Efficient Tests for an Autoregressive Unit Root," *Econometrica*, 64, 813-836.
- Enders, W. and B.-S. Lee, (1997), "Accounting for Real and Nominal Exchange Rate Movements In the Post-Bretton Woods Period," *Journal of International Money and Finance*, 16, 233-254.
- Engel, C., (2000), "Long-Run PPP May Not Hold After All," *Journal of International Economics*, 51, 243-273.
- Engel, C. and C.-J. Kim, (1999), "The Long-Run U.S./U.K. Real Exchange Rate," *Journal of Money, Credit, and Banking*, 31, 335-356.
- Evans, M. D. D. and J. R. Lothian, (1993), "The Responses of Exchange Rates to Permanent and Transitory Shocks under Floating Exchange Rates," *Journal of International Money and Finance*, 12, 563-586.
- Frankel, J. A. and A. K. Rose, (1996), "A Panel Project on Purchasing Power Parity: Mean Reversion Within and Between Countries," *Journal of International Economics*, 40, 209-224.
- Froot, K. A. and K. Rogoff, (1995), "Perspectives on PPP and Long-Run Real Exchange Rates," in, Grossman, G. and K. Rogoff, (ed.), *Handbook of International Economics*, Vol. III, North-Holland, Amsterdam.
- Johansen, S., (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, 59, 1551-1580.

- Johansen, S., (2000), "A Bartlett Correction Factor for Tests on the Cointegration Relations," *Econometric Theory*, 16, 740–778.
- Johansen, S. and K. Juselius, (1990), "Maximum Likelihood Estimation and Inference on Cointegration — With Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, 52, 169–210.
- King, R. G., C. I. Plosser, J. H. Stock and M. W. Watson, (1991), "Stochastic Trends and Economic Fluctuations," *American Economic Review*, 81, 819–840.
- Kuo, B.-S. and A. Mikkola, (1999), "Re-Examining Long-Run Purchasing Power Parity," *Journal of International Money and Finance*, 18, 251–266.
- Lastrapes, W. D., (1992), "Sources of Fluctuations in Real and Nominal Exchange Rates," *The Review of Economics and Statistics*, 74, 530–539.
- Lothian, J. R., (1997), "Multi-Country Evidence on the Behavior of Purchasing Power Parity under the Current Float," *Journal of International Money and Finance*, 16, 19–35.
- Lütkepohl, H. and H.-E. Reimers, (1992), "Impulse Response Analysis of Co-Integrated Systems," *Journal of Economic Dynamics and Control*, 16, 53–78.
- Mellander, E., A. Vredin and A. Warne, (1992), "Stochastic Trends and Economic Fluctuations in a Small Open Economy," *Journal of Applied Econometrics*, 7, 369–394.
- Obstfeld, M. and K. Rogoff, (1995a), "Exchange Rate Dynamics Redux," *Journal of Political Economy*, 103, 624–660.
- Obstfeld, M. and K. Rogoff, (1995b), "The Intertemporal Approach to the Current Account," in, Grossman, G. M. and K. Rogoff, (ed.), *Handbook of International Economics*, Vol., North-Holland, New York.
- Obstfeld, M. and K. Rogoff, (1996), *Foundations of International Macroeconomics*, MIT Press.
- Oh, K.-Y., (1996), "Purchasing Power Parity and Unit Root Tests Using Panel Data," *Journal of International Money and Finance*, 15, 405–18.
- Papell, D. H., (1997), "Searching for Stationarity: Purchasing Power Parity Under the Current Float," *Journal of International Economics*, 43, 313–332.
- Pesaran, M. H. and Y. Shin, (1998), "Generalized Impulse Response Analysis in Linear Multivariate Models," *Economics Letters*, 58, 17–29.
- Rogers, J. H., (1999), "Monetary Shocks and Real Exchange Rates," *Journal of International Economics*, 49, 269–288.

- Rogoff, K., (1992), “Traded Goods Consumption Smoothing and the Random Walk Behavior of the Real Exchange Rate,” *Bank of Japan Monetary and Economic Studies*, 10, 1–29.
- Rogoff, K., (1996), “The Purchasing Power Parity Puzzle,” *Journal of Economic Literature*, 34, 647–68.
- Stock, J. H. and M. W. Watson, (1988), “Testing for Common Trends,” *Journal of the American Statistical Association*, 83, 1097–1107.
- Taylor, M. P. and L. Sarno, (1998), “The Behavior of Real Exchange Rates During the Post–Bretton Woods Period,” *Journal of International Economics*, 46, 281–312.
- Wu, Y., (1996), “Are Real Exchange Rates Nonstationary? Evidence from a Panel–Data Test,” *Journal of Money, Credit, and Banking*, 28, 54–63.

## Appendix A. Identification for Common-Trend Decomposition

To determine  $F$ , we first derive  $C(L)$  and then find the CT representation of the VMA model. The basic analysis is based on King, Plosser, Stock and Watson (1991), Mellander, Vredin and Warne (1992), and Bergman (1996). Following Campbell and Shiller (1988), let

$$M = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}. \quad (\text{A.1})$$

Also, let  $\Gamma(L) = I - \sum_{j=1}^{p-1} \Gamma_j$ . Premultiplying both sides of the VEC model in (9) yields

$$M\Gamma(L)\Delta X_t = M\mu - M\Pi X_{t-p} + M\varepsilon_t. \quad (\text{A.2})$$

Define a stationary variable  $y_t = D_\perp(L)MX_t$ , where  $D_\perp$  is a diagonal matrix with its diagonal elements given by  $D_\perp(L)_{11} = 1 - L$  and  $D_\perp(L)_{22} = 1$ . This implies that  $y_t = \begin{bmatrix} (1-L)e_t & e_t - p_t \end{bmatrix}'$ . Then, we have

$$y_t = B^{-1}(L) \{M\mu + M\varepsilon_t\} \quad (\text{A.3})$$

with  $B(L) = M \{ \Gamma(L)M^{-1}D(L) + \alpha^*L^p \}$ , where  $\alpha^* = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix}' \begin{bmatrix} 0 & 1 \end{bmatrix}$  and  $D(L)$  is a diagonal matrix with its diagonal elements given by  $D(L)_{11} = 1$  and  $D(L)_{22} = 1 - L$ . Since  $\Delta X_t = M^{-1}D(L)y_t$ , comparing this with equation (18) yields

$$C(L) = M^{-1}D(L)B(L)^{-1}M. \quad (\text{A.4})$$

This relationship between estimates of  $B(L)$  and the moving average lag polynomial  $C(L)$  implies that

$$C(1) = \frac{1}{\det(B(1))} \begin{bmatrix} B(1)_{22} - B(1)_{12} & B(1)_{12} \\ B(1)_{22} - B(1)_{12} & B(1)_{12} \end{bmatrix} \quad (\text{A.5})$$

Moreover, the adjustment coefficients are given by  $\alpha_1 = B(1)_{12}$  and  $\alpha_2 = B(1)_{12} - B(1)_{22}$ . We next derive the CT component of the system. Equation (18) can be expressed as

$$\Delta X_t = \delta + C(1)\varepsilon_t + (1-L)C^*(L)\varepsilon_t \quad (\text{A.6})$$

by writing  $C(L)$  as  $C(1) + (1-L)C^*(L)$ . Hence, assuming that  $\varepsilon_0 = 0$ , we have

$$X_t = X_0 + \delta t + C(1) \sum_{i=1}^t \varepsilon_i + C^*(L)\varepsilon_t \quad (\text{A.7})$$

Comparing this with equation (20) gives  $\Phi\varphi_t = C(1)\varepsilon_t$ . Since  $\beta'\Phi = 0$  and  $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$ , we then have

$$\varphi_t = \det(B(1))^{-1} \begin{bmatrix} -\alpha_2 & \alpha_1 \end{bmatrix} \varepsilon_t = F_\varphi \varepsilon_t \quad (\text{A.8})$$

where  $F_\varphi = \det(B(1))^{-1} \begin{bmatrix} -\alpha_2 & \alpha_1 \end{bmatrix}$ . We follow Mellander, Vredin and Warne (1992) and find that

$$\psi_t = (\alpha'\Omega^{-1}\alpha)^{-1/2} \alpha'\Omega^{-1}\varepsilon_t = F_\psi \varepsilon_t. \quad (\text{A.9})$$

## Appendix B. Data Sources

*Consumer price index.* IMF's International Financial Statistics (IFS) CD-ROM

*Nominal exchange rate.* IFS CD-ROM.

*Industrial production.* OECD Main Economic Indicators.

Japan, the UK and the US: Industrial production, manufacturing.

Germany: Total industrial production.

*Employment.* OECD Main Economic Indicators.

Germany, Japan and the US: Employment in manufacturing.

UK: Quarterly data on total employment interpolated to monthly data.

*Money stock.*

Germany, Japan and the US: M1 taken from IFS CD-ROM.

UK: M2 from Bank of England.

*Long-term interest rate.* OECD Main Economic Indicators.

Germany: 7–15 year public sector bonds.

Japan: 10-year benchmark central government bonds.

UK: 10-year government bonds.

US: Government bonds over 10 years.



Table 1: Estimates of the Error–Correction Coefficients in the VEC Model

Coefficient	Germany	Japan	U.K.
$\alpha_1$	−0.018 (−1.966)**	−0.008 (−1.115)	−0.021 (−2.099)**
$\alpha_2$	0.002 (1.881)*	0.005 (2.909)**	0.005 (1.968)**

Notes:  $\alpha = [\alpha_1 \ \alpha_2]'$  is the coefficient vector associated with the error correction term in the VEC model in equation (17). For the reversion toward PPP to occur,  $\alpha_1$  should be negative and  $\alpha_2$  should be positive. The parentheses underneath individual coefficient estimates give the corresponding t–statistics. Statistical significance is indicated by a single asterisk (\*) for the 10% level and a double asterisk (\*\*) for the 5% level.

Table 2: Variance Decomposition of the Nominal Exchange Rate with Respect to Different Structural Innovations.

Horizon	Common–trend innovations			Transitory innovations		
	Germany	Japan	U.K.	Germany	Japan	U.K.
3	48.42	86.54	56.63	51.58	13.46	43.37
6	53.75	85.94	57.97	46.25	14.06	42.03
12	57.84	85.91	59.91	42.16	14.09	40.09
24	61.23	86.57	63.17	38.77	13.43	36.83
36	63.41	87.30	65.92	36.59	12.70	34.08
48	65.13	87.97	68.23	34.87	12.03	31.77
60	66.55	88.58	70.17	33.45	11.42	29.83
72	67.74	89.11	71.81	32.26	10.89	28.19
96	69.66	90.01	74.43	30.34	9.99	25.57
120	71.15	90.71	76.46	28.85	9.29	23.54

Note: The decomposition estimates are given in percentage.

Table 3: Variance Decomposition of the Relative Price with Respect to Different Structural Innovations.

Horizon	Common-trend innovations			Transitory innovations		
	Germany	Japan	U.K.	Germany	Japan	U.K.
3	64.38	13.59	43.28	35.62	86.41	56.72
6	67.56	19.15	47.98	32.44	80.85	52.02
12	73.07	27.39	55.56	26.93	72.61	44.44
24	80.92	43.65	67.56	19.08	56.35	32.44
36	86.00	57.23	75.87	14.00	42.77	24.13
48	89.34	67.40	81.49	10.66	32.60	18.51
60	91.59	74.73	85.33	8.41	25.27	14.67
72	93.17	79.98	88.02	6.83	20.02	11.98
96	95.14	86.57	91.40	4.86	13.43	8.60
120	96.27	90.28	93.37	3.73	9.72	6.63

Note: The decomposition estimates are given in percentage.

Table 4: Variance Decomposition of the Real Exchange Rate with Respect to Different Structural Innovations.

Horizon	Common-trend innovations			Transitory innovations		
	Germany	Japan	U.K.	Germany	Japan	U.K.
3	34.52	70.81	32.93	65.48	29.19	67.07
6	38.26	70.99	32.79	61.74	29.01	67.21
12	40.74	71.06	32.64	59.26	28.94	67.36
24	41.89	71.09	32.57	58.11	28.91	67.43
36	42.21	71.09	32.56	57.79	28.91	67.44
48	42.33	71.10	32.55	57.67	28.90	67.45
60	42.39	71.10	32.55	57.61	28.90	67.45
72	42.42	71.10	32.55	57.58	28.90	67.45
96	42.44	71.10	32.55	57.56	28.90	67.45
120	42.44	71.10	32.55	57.56	28.90	67.45

Note: The decomposition estimates are given in percentage.

Table 5: Causality Tests for the Macroeconomic Sources of the Common-Trend and Transitory Innovations.

Macroeconomic variable		Impact on common-trend innovations			Impact on transitory innovations		
		Germany	Japan	U.K.	Germany	Japan	U.K.
Productivity	Wald	4.78	5.10	0.08	2.09	0.43	0.40
	p-value	(0.09)	(0.07)	(0.96)	(0.35)	(0.80)	(0.82)
Money supply	Wald	8.00	4.95	4.57	1.96	3.68	7.97
	p-value	(0.02)	(0.08)	(0.10)	(0.38)	(0.16)	(0.02)
Interest rate	Wald	0.08	1.02	1.19	5.36	0.44	3.18
	p-value	(0.96)	(0.60)	(0.55)	(0.07)	(0.80)	(0.20)

Notes: The test results are based on estimation of the VARMA model under equations (29) and (30), with  $\Delta Y_t$  including three different macroeconomics variables: changes in labor productivity differentials, relative money supply, and nominal interest rate differentials. The Wald test statistics presented above examine the null hypothesis of no causal effects (i.e., the hypothesis that all the corresponding coefficients associated with the relevant variable are zero). Their corresponding p-values are given in parentheses underneath individual test statistics.

Figure 1: The common-trend component versus the nominal exchange rate, the relative price, and the real exchange rate.

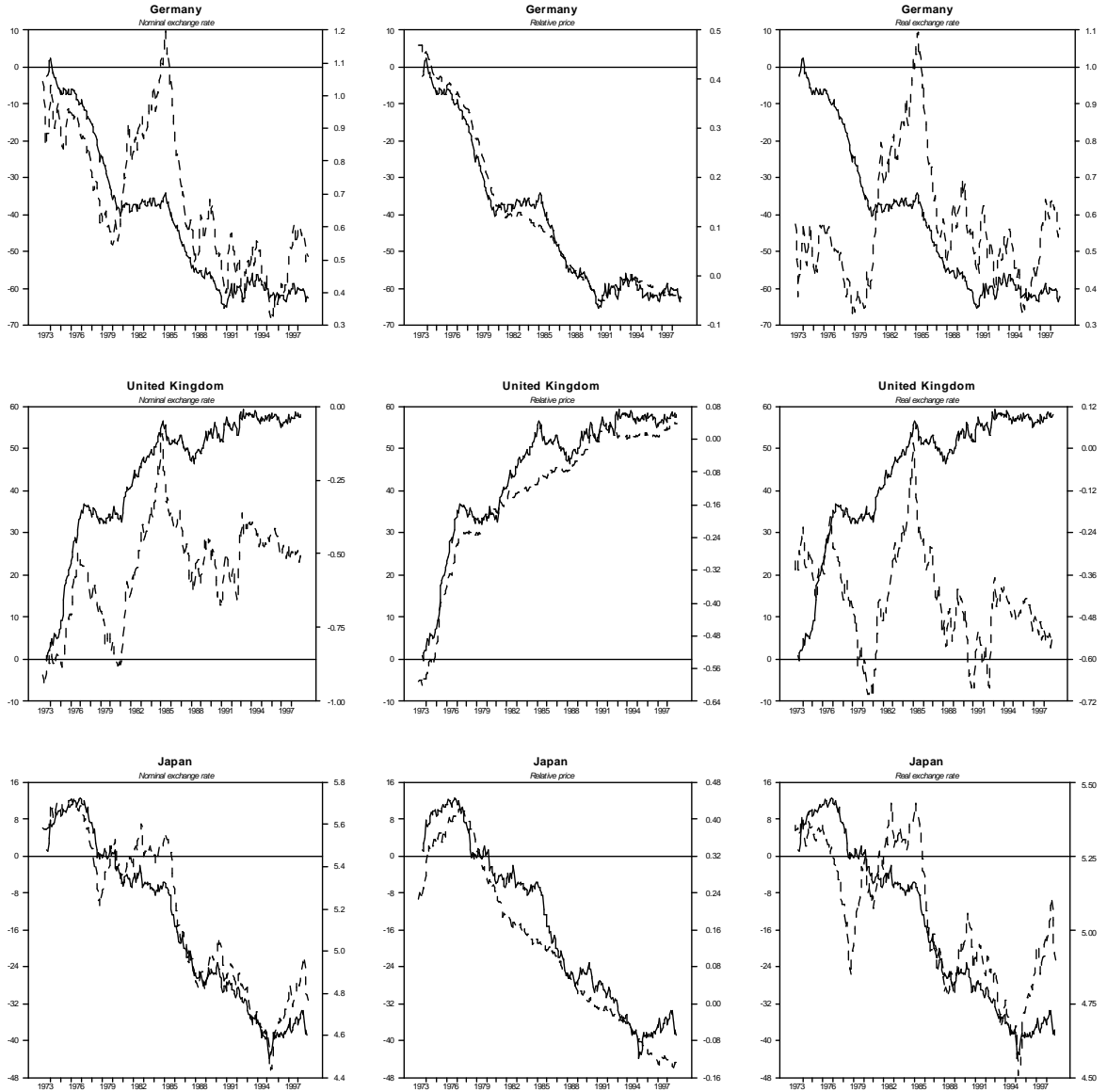
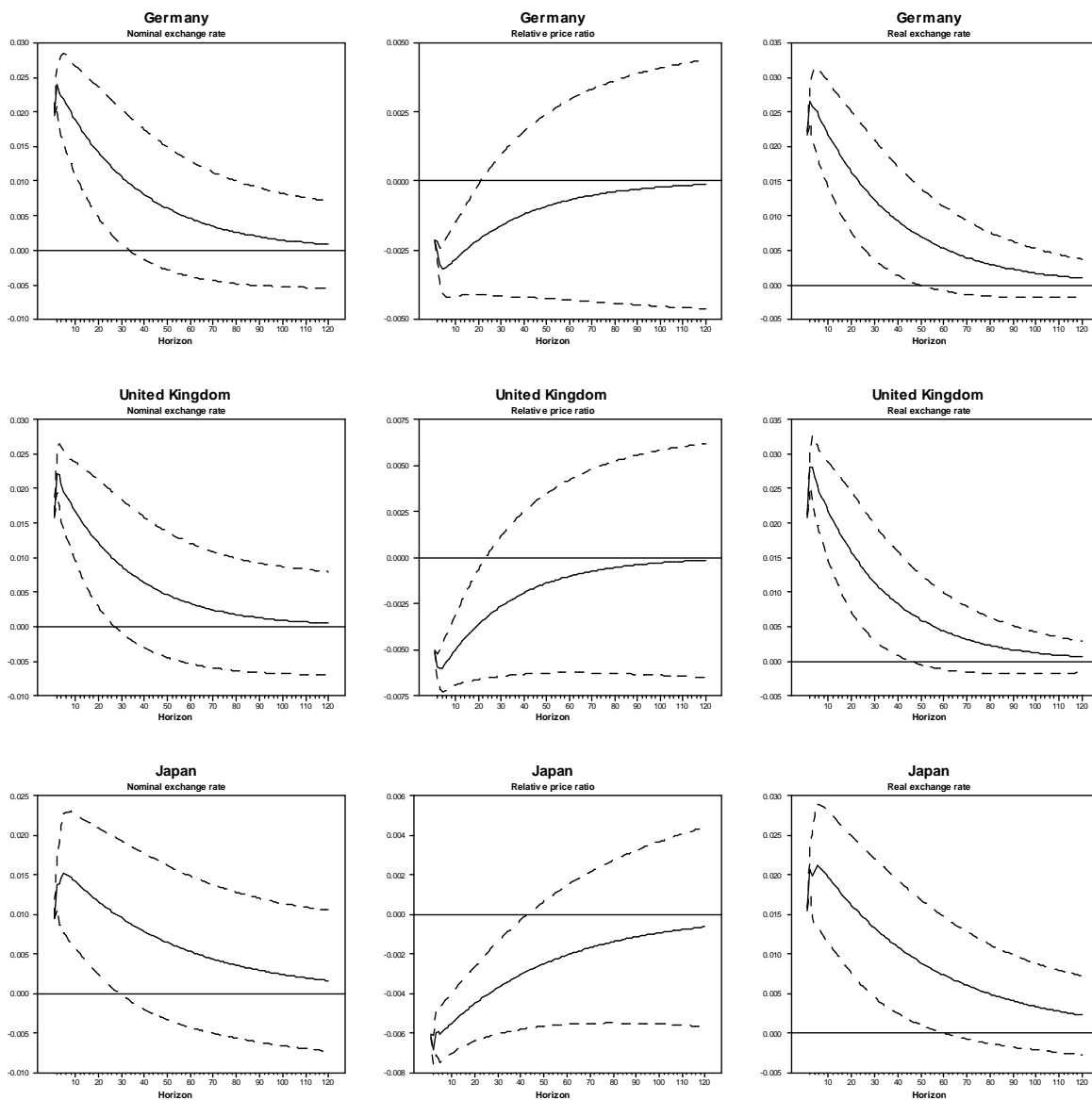
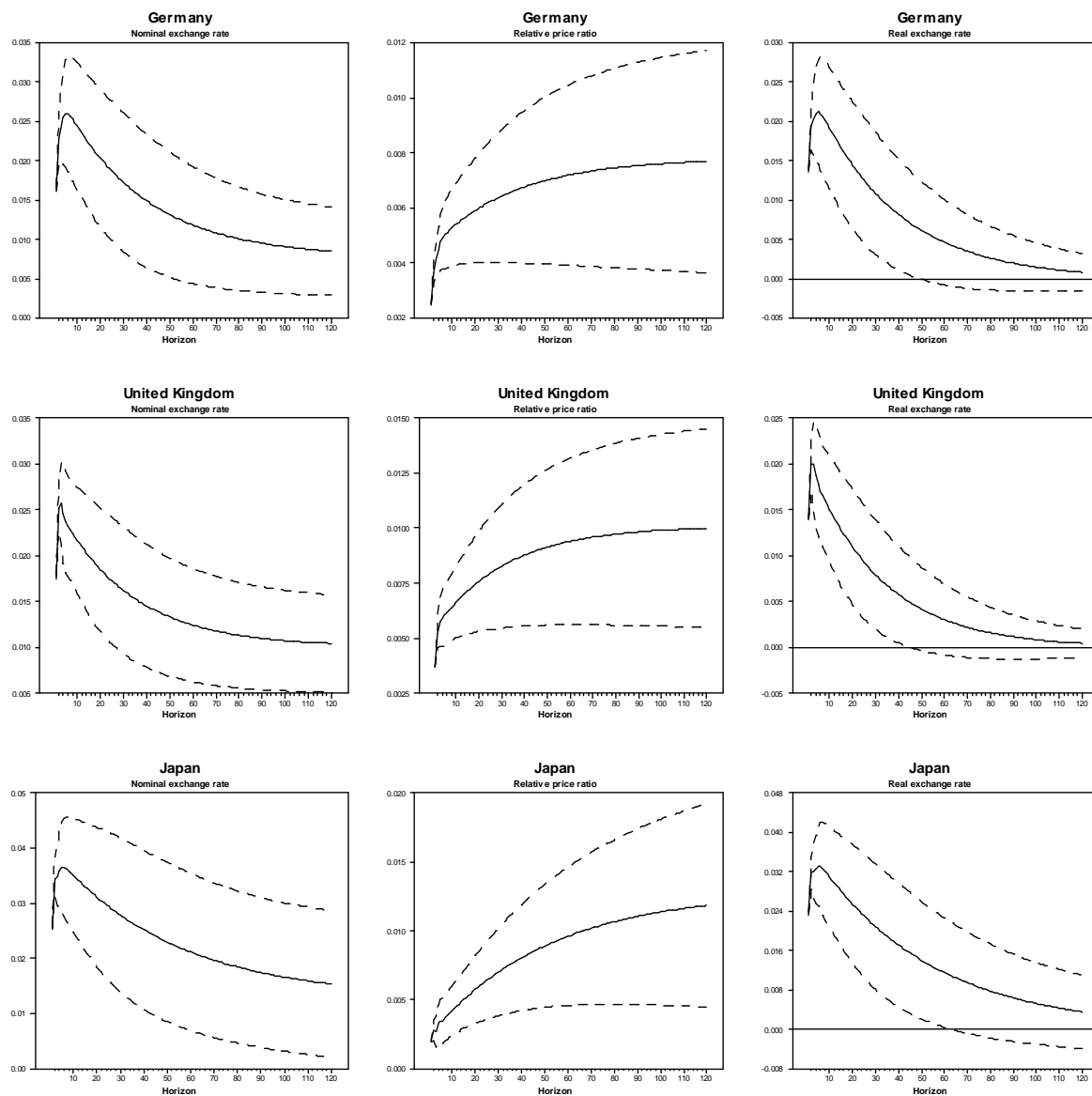


Figure 2: Impulse responses of the nominal exchange rate, the relative price, and the real exchange rate to transitory innovations.



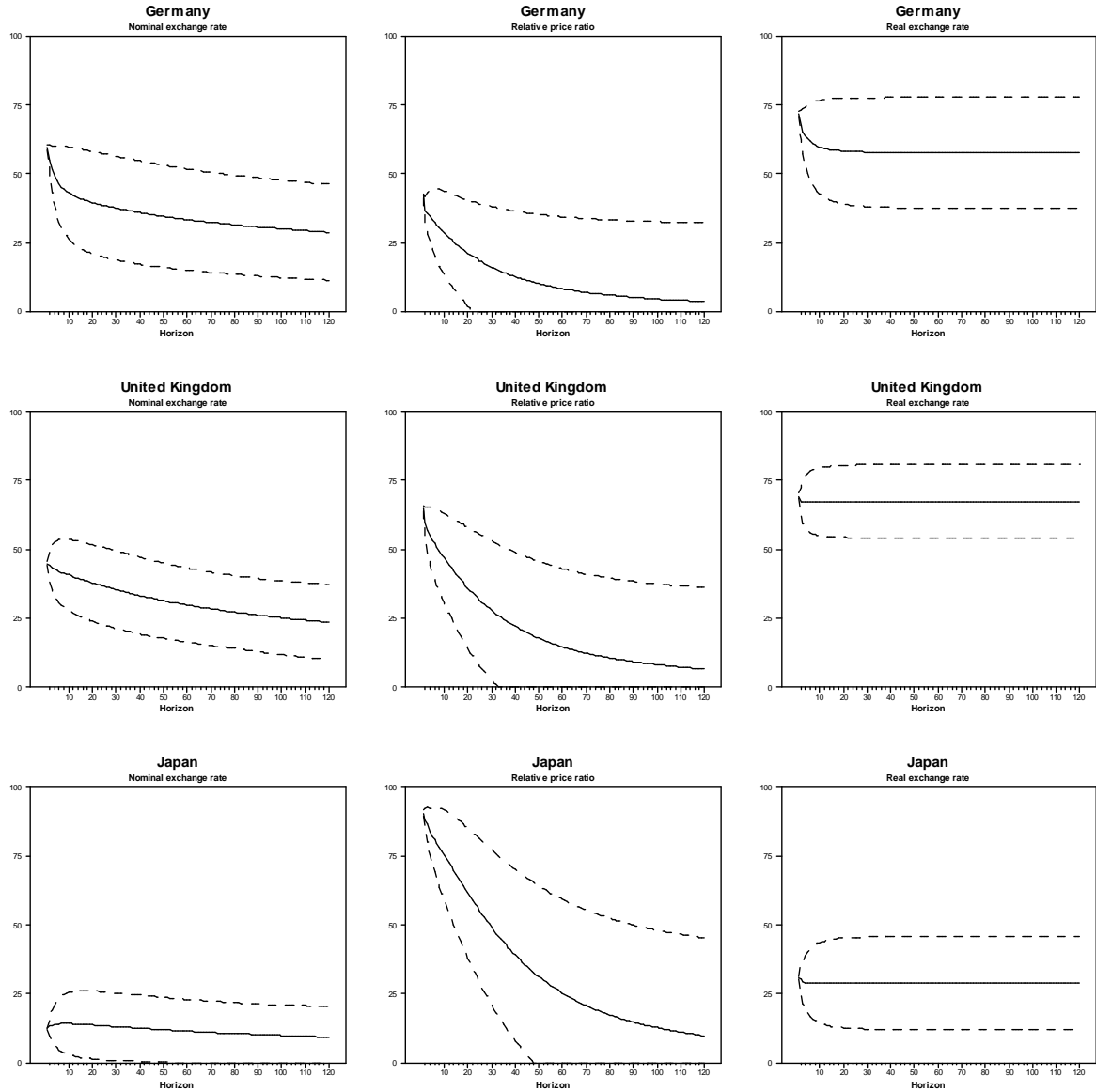
Note: The 95% confidence bands (upper and lower bands) for the impulse response estimates are shown by broken lines.

Figure 3: Impulse responses of the nominal exchange rate, the relative price, and the real exchange rate to common-trend innovations.



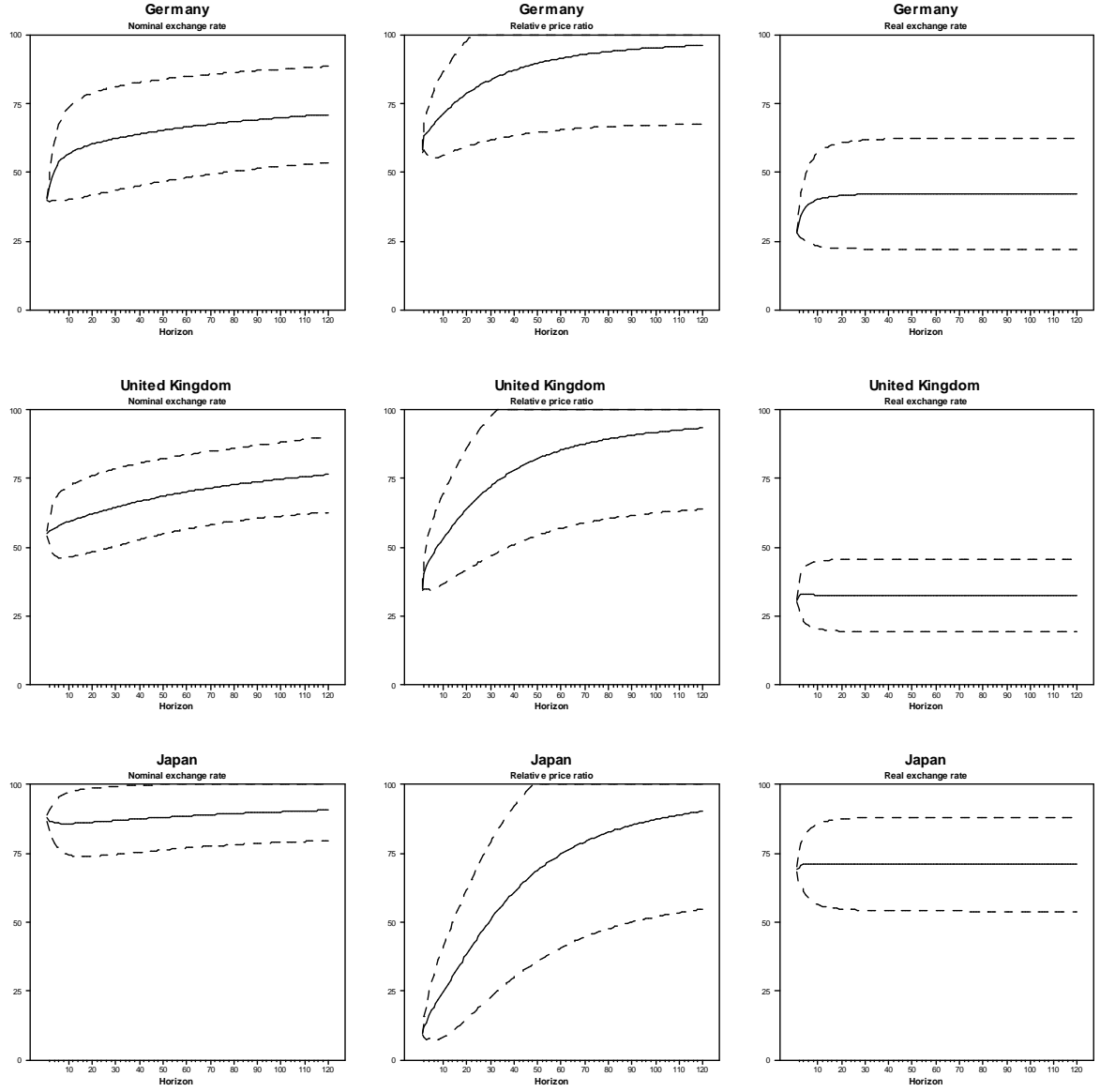
Note: The 95% confidence bands (upper and lower bands) for the impulse response estimates are shown by broken lines.

Figure 4: The proportion of forecast error variance of the different variables explained by transitory innovations.



Note: The 95% confidence bands (upper and lower bands) for the impulse response estimates are shown by broken lines.

Figure 5: The proportion of forecast error variance of the different variables explained by common-trends innovations.



Note: The 95% confidence bands (upper and lower bands) for the impulse response estimates are shown by broken lines.